$$\begin{cases} 5x_1 + x_2 + x_3 + x_4 = 4 \\ x_1 + 7x_2 + x_3 + x_4 = 12 \\ x_1 + x_2 + 6x_3 + x_4 = -5 \\ x_1 + x_2 + x_3 + 4x_4 = -6 \end{cases}$$

Rewrite the system in matrix form and solve it by Gaussian Elimination (Gauss-Jordan elimination)

$$\left(\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 1 & 1 & 1 & 4 & -6 \end{array}\right)$$

$R_1 / 5 \rightarrow R_1$ (divide the 1 row by 5)

$$\left(\begin{array}{cccc|c} \mathbf{1} & \mathbf{0.2} & \mathbf{0.2} & \mathbf{0.2} & \mathbf{0.8} \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 1 & 1 & 1 & 4 & -6 \end{array}\right)$$

$R_2 - 1\ R_1 \rightarrow R_2$ (multiply 1 row by 1 and subtract it from 2 row); $R_3 - 1\ R_1 \rightarrow R_3$ (multiply 1 row by 1 and subtract it from 3 row); $R_4 - 1\ R_1 \rightarrow R_4$ (multiply 1 row by 1 and subtract it from 4 row)

$$\left(\begin{array}{cccc|c} 1 & 0.2 & 0.2 & 0.2 & 0.8 \\ \mathbf{0} & \mathbf{6.8} & \mathbf{0.8} & \mathbf{0.8} & \mathbf{11.2} \\ \mathbf{0} & \mathbf{0.8} & \mathbf{5.8} & \mathbf{0.8} & \mathbf{-5.8} \\ \mathbf{0} & \mathbf{0.8} & \mathbf{0.8} & \mathbf{3.8} & \mathbf{-6.8} \end{array}\right)$$

$R_2 / 6.8 \rightarrow R_2$ (divide the 2 row by 6.8)

$$\left(\begin{array}{cccc|c} 1 & 0.2 & 0.2 & 0.2 & 0.8 \\ \mathbf{0} & \mathbf{1} & \frac{\mathbf{2}}{\mathbf{17}} & \frac{\mathbf{2}}{\mathbf{17}} & \frac{\mathbf{28}}{\mathbf{17}} \\ 0 & 0.8 & 5.8 & 0.8 & -5.8 \\ 0 & 0.8 & 0.8 & 3.8 & -6.8 \end{array}\right)$$

$R_1 - 0.2\ R_2 \rightarrow R_1$ (multiply 2 row by 0.2 and subtract it from 1 row); $R_3 - 0.8\ R_2 \rightarrow R_3$ (multiply 2 row by 0.8 and subtract it from 3 row); $R_4 - 0.8\ R_2 \rightarrow R_4$ (multiply 2 row by 0.8 and subtract it from 4 row)

$$\left(\begin{array}{cccc|c} \mathbf{1} & \mathbf{0} & \frac{\mathbf{3}}{\mathbf{17}} & \frac{\mathbf{3}}{\mathbf{17}} & \frac{\mathbf{8}}{\mathbf{17}} \\ 0 & 1 & \frac{2}{17} & \frac{2}{17} & \frac{28}{17} \\ \mathbf{0} & \mathbf{0} & \frac{\mathbf{97}}{\mathbf{17}} & \frac{\mathbf{12}}{\mathbf{17}} & -\frac{\mathbf{121}}{\mathbf{17}} \\ \mathbf{0} & \mathbf{0} & \frac{\mathbf{12}}{\mathbf{17}} & \frac{\mathbf{63}}{\mathbf{17}} & -\frac{\mathbf{138}}{\mathbf{17}} \end{array}\right)$$

$R_3 / \frac{97}{17} \rightarrow R_3$ (divide the 3 row by $\frac{97}{17}$)

$$\left(\begin{array}{cccc|c} 1 & 0 & \frac{3}{17} & \frac{3}{17} & \frac{8}{17} \\ 0 & 1 & \frac{2}{17} & \frac{2}{17} & \frac{28}{17} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{\mathbf{12}}{\mathbf{97}} & -\frac{\mathbf{121}}{\mathbf{97}} \\ 0 & 0 & \frac{12}{17} & \frac{63}{17} & -\frac{138}{17} \end{array}\right)$$

$R_1 - \frac{3}{17} R_3 \rightarrow R_1$ (multiply 3 row by $\frac{3}{17}$ and subtract it from 1 row); $R_2 - \frac{2}{17} R_3 \rightarrow R_2$ (multiply 3 row by $\frac{2}{17}$ and subtract it from 2 row); $R_4 - \frac{12}{17} R_3 \rightarrow R_4$ (multiply 3 row by $\frac{12}{17}$ and subtract it from 4 row)

$$\left(\begin{array}{cccc|c} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{\frac{15}{97}} & \mathbf{\frac{67}{97}} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{\frac{10}{97}} & \mathbf{\frac{174}{97}} \\ 0 & 0 & 1 & \frac{12}{97} & -\frac{121}{97} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\frac{351}{97}} & \mathbf{-\frac{702}{97}} \end{array}\right)$$

$R_4 / \frac{351}{97} \rightarrow R_4$ (divide the 4 row by $\frac{351}{97}$)

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{15}{97} & \frac{67}{97} \\ 0 & 1 & 0 & \frac{10}{97} & \frac{174}{97} \\ 0 & 0 & 1 & \frac{12}{97} & -\frac{121}{97} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-2} \end{array}\right)$$

$R_1 - \frac{15}{97} R_4 \rightarrow R_1$ (multiply 4 row by $\frac{15}{97}$ and subtract it from 1 row); $R_2 - \frac{10}{97} R_4 \rightarrow R_2$ (multiply 4 row by $\frac{10}{97}$ and subtract it from 2 row); $R_3 - \frac{12}{97} R_4 \rightarrow R_3$ (multiply 4 row by $\frac{12}{97}$ and subtract it from 3 row)

$$\left(\begin{array}{cccc|c} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{2} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{-1} \\ 0 & 0 & 0 & 1 & -2 \end{array}\right)$$

$$\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = -1 \\ x_4 = -2 \end{cases}$$

Make a check:

$5 \cdot 1 + 2 + (-1) + (-2) = 5 + 2 - 1 - 2 = 4$
$1 + 7 \cdot 2 + (-1) + (-2) = 1 + 14 - 1 - 2 = 12$
$1 + 2 + 6 \cdot (-1) + (-2) = 1 + 2 - 6 - 2 = -5$
$1 + 2 + (-1) + 4 \cdot (-2) = 1 + 2 - 1 - 8 = -6$

Check completed successfully.

Answer:

$$\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = -1 \\ x_4 = -2 \end{cases}$$