

Q Solve the following problem using DPP

Maximize  $Z = 2x_1 + 5x_2$

Subject to constraints:

$$2x_1 + x_2 \leq 430$$

$$2x_2 \leq 460$$

$$x_1, x_2 \geq 0$$

Sol:

State = 2 (no. of constraints)

Stage = 2 (no. of variables)

Obj. function =  $Z$  max

Assume 2 resources  $\rightarrow b_1, b_2$

$F_i(b_1, b_2)$

$i \rightarrow$  stage

Stage 1:

$i = 1$

$$F_1(b_1, b_2) = \text{Max } 2x_1$$

$$0 \leq x_1 \leq b$$

To calculate  $b \text{ Min } \left\{ \frac{b_1}{c_{x_1}}, \frac{b_2}{c_{x_2}} \right\}$

$$= \text{Min} \left\{ \frac{430}{2}, \frac{460}{0} \right\}$$

$$= \text{Min} \{ 215, \infty \}$$

$$b = 215$$

$$F_1(b_1, b_2) = \text{Max } 2x_1$$

$$0 \leq x_1 \leq 215$$



$$\text{constraint ①} \rightarrow x_1 = \frac{430 - x_2}{2}$$

$$\text{constraint ②} \rightarrow x_1 = 0$$

$$\text{Min} \{ x_1, x_2 \} \Rightarrow \text{Min} \left\{ \frac{430 - x_2}{2}, 0 \right\}$$

$$x_1 = \frac{430 - x_2}{2}$$

$$f_1(430, 460) = 2 \text{ Min} \left\{ \frac{430 - x_2}{2} \right\}$$

Stage 2:  $i = 2$

$$f_2(b_1, b_2) = \text{Max } 2x_1 + 5x_2$$

$$= \text{Max } 5x_2 + 2 \left[ \text{Min} \left( \frac{430 - x_2}{2} \right) \right]$$

• To calculate  $b$  for  $x_2$   $b = \text{Min} \left\{ \frac{b_1}{c_{x_2}}, \frac{b_2}{c_{x_2}} \right\}$

$$b = \text{Min} \left\{ \frac{430}{1}, \frac{460}{6} \right\} = \text{Min} \{ 430, 230 \}$$

$$\boxed{b = 230}$$

$$f_2(430, 460) = \text{Max } 5x_2 + 2 \left[ \text{Min} \left( \frac{430 - x_2}{2} \right) \right]$$

$$0 \leq x_2 \leq 230$$

To calculate  $\left[ \text{Min} \left( \frac{430 - x_2}{2} \right) \right]$

$$\text{Min} \begin{cases} 430 & \text{when } x_2 = 0 \\ 100 & \text{when } x_2 = 230 \end{cases}$$



$$x_2 = 230$$

$$x_1 = \min \left\{ \frac{430 - x_2}{2} \right\}$$

$$x_1 = 100$$

$$Z = 2x_1 + 5x_2$$

$$Z = 2(100) + 5(230)$$

$$Z = 200 + 1150$$

$$Z = 1350$$

Ans