

$$Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

$$s.t. \quad x_1 + x_2 + x_3 = 20$$

$$x_1, x_2, x_3 \geq 0$$

$$\Rightarrow L(x, \lambda) = (2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100) + \lambda(x_1 + x_2 + x_3 - 20)$$

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 4x_1 + 10 + \lambda = 0 \quad \text{--- (i)}$$

$$\Rightarrow x_1 = \frac{-10 - \lambda}{4}$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 2x_2 + 8 + \lambda = 0 \quad \text{--- (ii)}$$

$$\Rightarrow x_2 = \frac{-8 - \lambda}{2}$$

$$\frac{\partial L}{\partial x_3} = 0 \Rightarrow (3 \times 2)x_3 + 6 + \lambda = 0 \quad \text{--- (iii)}$$

$$\Rightarrow x_3 = \frac{-6 - \lambda}{6}$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow x_1 + x_2 + x_3 - 20 = 0 \quad \text{--- (iv)}$$

Put  $x_1, x_2, x_3$  in eq (IV)

$$\Rightarrow \left( \frac{-10-\lambda}{4} \right) + \left( \frac{-8-\lambda}{2} \right) + \left( \frac{-6-\lambda}{6} \right) - 20 = 0$$

$$\Rightarrow \frac{-30-3\lambda - 48-6\lambda - 12-2\lambda}{12} - 20 = 0$$

$$\Rightarrow \frac{-90-11\lambda}{12} - 20 = 0$$

$$\Rightarrow -90-11\lambda - 240 = 0$$

$$\Rightarrow -330-11\lambda = 0$$

$$\Rightarrow \lambda = \frac{-330}{11}$$

$$x_1 = \frac{-10 + 330}{4} = \frac{-110 + 330}{44}$$

$$\frac{220}{44} = 5$$

$$x_2 = \frac{-8 + 330}{2} = \frac{-88 + 330}{22} = \frac{242}{22} = 11$$

$$x_3 = \frac{-6 + 330}{6} = \frac{-66 + 330}{66} = \frac{264}{66} = 4$$

$$L(x, \lambda) = (5, 11, 4; -20)$$

$$H^B = \begin{bmatrix} 0 & 0 \\ 0^T & V \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 6 \end{bmatrix}$$

$$D_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 4 & 0 \\ 1 & 0 & 2 \end{vmatrix} = -1(2) + 1(-4) \\ = -2 - 4 = -6$$

$$D_4 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 6 \end{vmatrix} = -44$$

So, stationary point is our minimum point. i.e. fn is convex function.