

$$f(x) = z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$$

$$s.t = x_1 + x_2 + x_3 = 7$$

$$g(x) = x_1 + x_2 + x_3 - 7 = 0$$

$$L(x, \lambda) = (x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3) + \lambda (x_1 + x_2 + x_3 - 7)$$

$$\frac{\partial L}{\partial x_1} = 2x_1 - 10 + \lambda = 0$$

$$\Rightarrow x_1 = \frac{10 - \lambda}{2} \quad \text{--- (i)}$$

$$\frac{\partial L}{\partial x_2} = 2x_2 - 6 + \lambda = 0$$

$$\Rightarrow x_2 = \frac{6 - \lambda}{2} \quad \text{--- (ii)}$$

$$\frac{\partial L}{\partial x_3} = 2x_3 - 4 + \lambda = 0$$

$$\Rightarrow x_3 = \frac{4 - \lambda}{2} \quad \text{--- (iii)}$$

$$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + x_3 - 7 = 0 \quad \text{--- (iv)}$$

Now, from eq (i), (ii), (iii) & (iv) we have,

$$f(x_1, x_2, x_3) = (4, 2, 1) \quad \text{and } \lambda = 2$$

$$HB: \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$D_1 = 2 > 0$$

$$D_2 = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0$$

$$D_3 = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 8 > 0$$

Objective function is Convex So, Minimum Occur.

$$Z = (4)^2 - 10(4) + (2)^2 - 6 \times (2) + (1)^2 - 4$$

$$= 16 - 40 + 4 - 12 + 1 - 4$$

$$= 21 - 56 = -35 \rightarrow \text{Minimum Value}$$