Max $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$

s/c $x_1 + x_2 = 2$

$x_1, x_2 \geq 0$

Sol/n By: Lagrangian Multiplier

$L = f - \lambda H$

$f(x) = 4x_1 + 8x_2 - x_1^2 - x_2^2$

$H = x_1 + x_2 - 2$

Now,

$L = (4x_1 + 8x_2 - x_1^2 - x_2^2) - \lambda(x_1 + x_2 - 2)$

$\frac{\partial L}{\partial x_1} = 4 - 2x_1 - \lambda = 0$ — (i)

$\frac{\partial L}{\partial x_2} = 8 - 2x_2 - \lambda = 0$ — (ii)

$\frac{\partial L}{\partial \lambda} = 0 - (x_1 + x_2 - 2) = 0$ — (iii)

Now, eq(i) = eq(ii)

$4 - 2x_1 = 8 - 2x_2$

$-2x_1 + 2x_2 = 8 - 4$

$(x_2 - x_1) = 2$ — (iv)

Now, eq (iv) & eq (iii)

$$\cancel{x_2} - x_1 = 2$$
$$-x_1 - \cancel{x_2} = -2$$
$$+ \overline{}$$
$$-2x_1 = 0$$
$$\boxed{x_1 = 0}$$

Now,

$$x_2 - x_1 = 2$$

$\boxed{x_2 = 2}$ and $\boxed{\lambda = 4}$

Now,

$$\frac{\partial^2 L}{\partial x_1^2} = -2 \; ; \; \frac{\partial^2 L}{\partial x_2^2} = -2$$

$$\frac{\partial^2 L}{\partial x_1 \partial x_2} = 0 \; ; \; \frac{\partial h}{\partial x_1} = 1 \; ; \; \frac{\partial h}{\partial x_2} = 1$$

$$\Delta = \begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{vmatrix} = 4$$

o, Δ is positive & max at $x_0(0, 2)$

$$Z = 4x_1 - x_1^2 + 8x_2 - x_2^2$$
$$= 4(0) - (0) + 8(2) - (2)^2$$
$$= +16 - 4 = 12$$