FORMULA:

EXECUTION TIME ,SPEED UP RATIO, CPI FOR PROGRAM /MACHINE

→ $$\text{Clock cycle} = \frac{1}{\text{clockrate}} \quad \text{or,} \quad \frac{1}{\text{clock frequency}}.$$

→ $$\text{Speed Up} = \frac{\text{Performance X}}{\text{Performance Y}}$$

or,

$$\text{Speed Up} = \frac{\text{Old Executione Time}}{\text{New Exec. Time}}$$

→ $$\text{Execution Time} = \text{clock cycle} \times \sum_{i=1}^{n} CPI_i \times I_i$$

{ where,
CPI = Average number of clocks
I = no. of Times instructions is executed. }

or,

$$E_T = IC \times CPI \times \frac{1}{\text{clock Rate}}$$ ⟶ or clock frequency.

→ $$CPI_{(overall)} = (CPI_1 \times f_1) + (CPI_2 \times f_2) \dots + (CPI_n \times f_n)$$

{ where f is frequency in fraction not in percentage }

RELATED QUESTION BASED ON FORMULA :

1. A processor having a clock cycle time of 0.25 nsec will have a clock rate of ____MHz.

   Solution: Clock cycle time *C* is the reciprocal of the clock rate *f*:

   C = 1 / f

   f= 1/C = 1/0.25ns = 4 GHz or 4000 MHz

FORMULA USED:

Clock time = 1/clock rate

Or, Clock time = 1/frequency

2. The performance of machine A is 10 times the performance of machine B, when running the same program. We say that machine A is __ times faster than machine B when running the same program.

Solution: $Speedup = \frac{PerformanceX}{PerformanceY} = \frac{10}{1} = 10$

. Consider a program whose instruction count is 50,000, average CPI is 2.2, and clock rate is 1.9 GHz. Suppose we use a new compiler on the same program for which the new instruction count is 40,000, and new CPI is 3.1, which is running on a faster machine with clock rate 2.5 GHz. The speedup achieved will be ____.

Solution: IC_1=50,000

CPI_1=2.2

Clock $rate_1$ = 1.9 GHz

$$OldExecutionTime_1 = IC_1 \times CPI_1 \times \frac{1}{clockrate_1}$$

$$= 50000 \times 2.2 \times \frac{1}{1.9 \times 10^9} \approx 5.789 \times 10^{-5} sec$$

IC_2=40,000
CPI_2=3.1
Clock $rate_2$ = 2.5 GHz

$$NewExecutionTime_2 = IC_2 \times CPI_2 \times \frac{1}{clockrate_2}$$
$$= 40000 \times 3.1 \times \frac{1}{2.5 \times 10^9} \approx 4.96 \times 10^{-5} sec$$

$$\text{Speedup} = \frac{OldExecutionTime_1}{NewExecutionTime_2} = \frac{5.789 \times 10^{-5} sec}{4.96 \times 10^{-5} sec} \approx 1.167$$

NOTE: **PERFORMANCE = 1/ EXECUTION TIME** -----EQ(1)

SO **SPEED UP = PERFORMACE *X* / PERFORMANCE *Y***

OR, **SPEED UP = EXECUTION TIME OF *Y* / EXECUTION TIME OF *X*** [FROM EQ(1)]

. A program is running on a machine which has a total of 500 instructions, average cycles per instruction for the program is 2.5, and CPU clock rate is 1.78 GHz. The execution time of the program will be ______ second.

Solution: Instruction Count = 500
CPI= 2.5
Clock Rate =1.78 GHz
Clock Cycle Time = 1/Clock rate

$$Execution_{Time} = IC \times CPI \times \frac{1}{clockrate} = 500 \times 2.5 \times \frac{1}{1.78 \times 10^9} \approx 702.25 \times 10^{-9}$$

Suppose for a RISC ISA implementation, there are four instruction types LOAD, STORE, ALU and BRANCH with relative frequencies of 25%, 3%, 50% and 22% respectively, and CPI values of 5, 2.5, 1 and 2.2 respectively. The overall CPI will be 2.309.

Solutions:

$$CPI_{overall} = (0.25 \times 5) + (0.03 \times 2.5) + (0.5 \times 1) + (0.22 \times 2.2) = 2.309$$

$$CPI_{OVERALL} = (CPI_1 \times F_1) + (CP_2 \times F_2) \; \ldots + (CPI_N \times F_N)$$

FORMULA:

⇒ MIPS Rating = $\frac{\text{Instruction Count (IC)}}{\text{Execution Time} \times 10^6}$

or,

$\frac{\text{Clock Rate (in Mhz)}}{\text{CPI}}$

⇒ Weighted Arithmetic Mean (WAM) = $\sum_{i=1}^{n} \frac{w_i t_i}{n}$

{ where, w = weights of Programs
t = time to run ; n = no. of Program }

⇒ Geometric Mean $\left(\frac{A}{B}\right)$ = $\frac{\text{Geometric Mean A}}{\text{Geometric Mean B}}$

⇒ Max SpeedUp = $\frac{1}{1-F}$ { where F = Speed of fraction }

⇒ Data Transfer Rate = $\frac{\text{Amount of Data}}{\text{Transfer Time}}$

{ Data Transfer Can be in bits per Second (b/s); bytes per second (B/s) ; Kilobyte per second (KB/s) ; megabyte per second (MB/s) so on. }

Consider a processor having four types of instruction classes, A, B, C and D, with the corresponding CPI values 1.1, 1.7, 2.8 and 3.5 respectively. The processor runs at a clock rate of 2 GHz. For a given program, the instruction counts for the four types of instructions are 20, 15, 12 and 5 million respectively. The MIPS rating of the processor for this program will be

Solution:

	A	B	C	D
CPI	1.1	1.7	2.8	3.5
IC	20	15	12	5

$$CPI = \frac{(20 \times 1.1) + (15 \times 1.7) + (12 \times 2.8) + (5 \times 3.5)}{20 + 15 + 12 + 5} = \frac{98.6}{52}$$
$$\approx 1.896$$

MIPS = Clock Rate (**in MHz**)/ (CPI) = $\frac{2000}{1.896} \approx 1054.8$ MIPS

ALSO ,MIPS RATING = INSTRUCTION COUNT / EXECUTION TIME x 10^6

There are three computers A, B and C. A program P1 takes time 5, 20 and 100 respectively to run on the three computers. Similarly, another program P2 takes times 750, 75 and 15 respectively to run on the same three computers. Which computer is the fastest based on weighted average mean assuming the weights of programs P1 and P2 to be 40% and 60% respectively?

a. Computer A
b. Computer B
c. Computer C
d. Cannot say.

Solution: (c)

	Computer A	Computer B	Computer C
P1	5	20	100
P2	750	75	15

$$Weighted\ Arithmetic\ Mean(WAM) = \sum_{i=1}^{n} \frac{w_i t_i}{n}$$

$$WAM\ for\ Computer\ A = \frac{(0.4 \times 5) + (0.6 \times 750)}{2} = 226$$

$$WAM\ for\ Computer\ B = \frac{(0.4 \times 20) + (0.6 \times 75)}{2} = 26.5$$

$$WAM\ for\ Computer\ C = \frac{(0.4 \times 100) + (0.6 \times 15)}{2} = 24.5$$

Which of the following statements are true?

a. Geometric means of normalized execution times are consistent.
b. Arithmetic means of normalized execution times are consistent.
c. Geometric means of normalized execution times are not consistent.
d. Arithmetic means of normalized execution times are not consistent.

Solution: ((a) and (d)) The geometric mean is independent of data series is used in normalization due to the property:

$$\frac{Geometric\ mean(A)}{Geometric\ mean(B)} = Geometric\ mean\left(\frac{A}{B}\right)$$

Suppose we are enhancing the speed of a fraction F of a given computation. If F = 0.55, the maximum speed up that can be attained is

Solution:

$$\text{Maximum Speedup} = \frac{1}{1-F} = \frac{1}{1-0.55} = \frac{1}{0.45} = 2.22$$

Consider two alternatives for speeding up computation. In the first alternative, we make 20% of a program 90 times faster. In the second alternative, we make 95% of the program 15 times faster. The ratio of the speedups for the two cases will be

Solution:

$$\text{90X faster for 20\% of the program} = \frac{1}{(1-0.2)+\frac{0.2}{90}} = \frac{1}{0.8022} \approx 1.246$$

$$\text{15X faster for 95\% of the program} = \frac{1}{(1-0.95)+\frac{0.95}{15}} = \frac{1}{0.11333} \approx 8.8235$$

Ratios of speedup is 1.246: 8.8235$\approx$ 1: 7 = 0.143

DATA TRANSFER RATE or SPEED = AMOUNT OF DATA / TRANSFER TIME

EXAMPLE:

For instance, say you transferred 100 GB at a rate of 7 MB/s. First, convert Gigabyte to Megabyte so you're working with the same units in every part of the equation. 100 x 1,024 = 102400. So, you transferred 102400 Megabyte at a rate of 7 MB/s. To solve for T, divide 102400 by 7, which is 14628.57. Therefore, it took 14628.57 sec. Now convert this to hours, divide by 3,600, which is 4.07. In other words, it took 4.06 hrs to transfer 100 GB at a rate of 7 MB/s.

CONVERSION TABLE:

8b = 1B	
1024 B = 1 KB	60 s = 1 min
1024 kb = 1 MB	60 min = 1 h
1024 mb = 1 GB	
1024 gb = 1 TB	

Magnetic Disk in Computer Architecture-

QUESTION:

Consider a disk pack with the following specifications- 16 surfaces, 128 tracks per surface, 256 sectors per track and 512 bytes per sector.

Answer the following questions-

1. What is the capacity of disk pack?
2. What is the number of bits required to address the sector?
3. If the format overhead is 32 bytes per sector, what is the formatted disk space?
4. If the format overhead is 64 bytes per sector, how much amount of memory is lost due to formatting?
5. If the diameter of innermost track is 21 cm, what is the maximum recording density?
6. If the diameter of innermost track is 21 cm with 2 KB/cm, what is the capacity of one track?
7. If the disk is rotating at 3600 RPM, what is the data transfer rate?
8. If the disk system has rotational speed of 3000 RPM, what is the average access time with a seek time of 11.5 msec?

SOL:

Part-01: Capacity of Disk Pack-

Capacity of disk pack

= Total number of surfaces x Number of tracks per surface x Number of sectors per track x Number of bytes per sector

= 16 x 128 x 256 x 512 bytes

= 2^{28} bytes

= 256 MB

Part-02: Number of Bits Required To Address Sector-

Total number of sectors

= Total number of surfaces x Number of tracks per surface x Number of sectors per track

= 16 x 128 x 256 sectors

= 2^{19} sectors

Thus, Number of bits required to address the sector = 19 bits

Part-03: Formatted Disk Space-

Formatting overhead

= Total number of sectors x overhead per sector

= 2^{19} x 32 bytes

= 2^{19} x 2^5 bytes

= 2^{24} bytes

= 16 MB

Now, Formatted disk space

= Total disk space – Formatting overhead

= 256 MB – 16 MB

= 240 MB

Part-04: Formatting Overhead-

Amount of memory lost due to formatting

= Formatting overhead

= Total number of sectors x Overhead per sector

= 2^{19} x 64 bytes

= 2^{19} x 2^6 bytes

= 2^{25} bytes

= 32 MB

Part-05: Maximum Recording Density-

Storage capacity of a track

= Number of sectors per track x Number of bytes per sector

= 256 x 512 bytes

= 2^8 x 2^9 bytes

= 2^{17} bytes

= 128 KB

Circumference of innermost track

= 2 x π x radius

= π x diameter

= 3.14 x 21 cm

= 65.94 cm

Now, Maximum recording density

= Recording density of innermost track

= Capacity of a track / Circumference of innermost track

= 128 KB / 65.94 cm

= 1.94 KB/cm

Part-06: Capacity Of Track-

Circumference of innermost track

= 2 x π x radius

= π x diameter

= 3.14 x 21 cm

= 65.94 cm

Capacity of a track

= Storage density of the innermost track x Circumference of the innermost track

= 2 KB/cm x 65.94 cm

= 131.88 KB

≅ 132 KB

Part-07: Data Transfer Rate-

Number of rotations in one second

= (3600 / 60) rotations/sec

= 60 rotations/sec

Now, Data transfer rate

= Number of heads x Capacity of one track x Number of rotations in one second

= 16 x (256 x 512 bytes) x 60

= 2^4 x 2^8 x 2^9 x 60 bytes/sec

= 60 x 2^{21} bytes/sec

= 120 MBps

Part-08: Average Access Time-

Time taken for one full rotation

= (60 / 3000) sec

= (1 / 50) sec

= 0.02 sec

= 20 msec

Average rotational delay

= 1/2 x Time taken for one full rotation

= 1/2 x 20 msec

= 10 msec

Now, average access time

= Average seek time + Average rotational delay + Other factors

= 11.5 msec + 10 msec + 0

= 21.5 msec

Problem-02:

What is the average access time for transferring 512 bytes of data with the following specifications-

- Average seek time = 5 msec
- Disk rotation = 6000 RPM
- Data rate = 40 KB/sec
- Controller overhead = 0.1 msec

Time Taken For One Full Rotation-

Time taken for one full rotation

= (60 / 6000) sec

= (1 / 100) sec

= 0.01 sec

= 10 msec

Average Rotational Delay-

Average rotational delay

= 1/2 x Time taken for one full rotation

= 1/2 x 10 msec

= 5 msec

Transfer Time-

Transfer time

= (512 bytes / 40 KB) sec

= 0.0125 sec

= 12.5 msec

Average Access Time-

Average access time

= Average seek time + Average rotational delay + Transfer time + Controller overhead + Queuing delay

= 5 msec + 5 msec + 12.5 msec + 0.1 msec + 0

= 22.6 msec

Pipelining in Computer Architecture-

Question

Q// Given; → Four stage pipeline used (k = 4)
→ Delay of stages = {(60, 50, 90, 80) ns}
→ Latch Delay = 10 ns
→ Instruction (n) = 1

(i) [Pipeline Cycle Time / Execution Time = Maximum Delay + Latch/Register Delay.]

A/Q
Pipeline Cycle Time = max{60, 50, 90, 80} + 10ns

⇒ (90 + 10) = 100 ns {Choose max from delay set}

(ii) [Non-pipeline Execution Time For 1 Instruction = Sum of all Delay at stages × n]

A/Q.
Non-pipeline Execution Time = (60 + 50 + 90 + 80) = 280ns × 1
= 280ns

(iii) [Speed up Ration (Pipeline & non-pipeline) = Non-Pipeline Exe. Time / Pipeline Exe. Time]

[Also; speed Ratio = Performance of piplined processor / " " non-piplined "]

A/Q =
$$\text{Speed Ratio} = \frac{ET_{(\text{non-pipline})}}{ET_{(\text{pipline})}} = \frac{280\,ns}{100\,ns} = 2.8$$

(iv) [Total Execution Time For Pipeline of 1000 task = $\{(K-1)+n\} \times$ cycle Time]

A/Q

K = 4 stage   n = 1000 (Given 1000 instruction Task)

Cycle Time = max{60, 50, 90, 80} + 10 ns = 100 ns

Total E.T = $\{(4-1)+1000\} \times 100$

= 100300 ns

(v) [For Non-Pipeline of 1000 task Total E.T = (n × total cycle Time)]

A/Q Total E.T (non-pipeline) = 1000 × 280 = 280000 ns

(vi) [Efficiency = $\frac{\text{Speed}}{\text{stage}} = \frac{S}{K}$]

Note: Cycle per Instruction (CPI) for Ideal Pipeline is 1

Note: If clock rate is Given in Question in GHz i.e $\times 10^9$ then find clock Time by formula ⇒

$$\text{clock Time} = \frac{1}{\text{clock rate}}$$

Exa: clock rate = 4 GHz then

$$\text{clock time} = \frac{1}{4 \times 10^9} = 0.25 \times 10^{-9}$$

= 0.25 nanosec

Problem-03:

Consider a non-pipelined processor with a clock rate of 2.5 gigahertz and average cycles per instruction of 4. The same processor is upgraded to a pipelined processor with five stages but due to the internal pipeline delay, the clock speed is reduced to 2 gigahertz. Assume there are no stalls in the pipeline. The speed up achieved in this pipelined processor is-

SOL:

Cycle Time in Non-Pipelined Processor-

Frequency of the clock = 2.5 gigahertz

Cycle time

= 1 / frequency

= 1 / (2.5 gigahertz)

= 1 / (2.5 x 10^9 hertz)

= 0.4 ns

Non-Pipeline Execution Time-

Non-pipeline execution time to process 1 instruction

= Number of clock cycles taken to execute one instruction

= 4 clock cycles

= 4 x 0.4 ns

= 1.6 ns

Cycle Time in Pipelined Processor-

Frequency of the clock = 2 gigahertz

Cycle time

= 1 / frequency

= 1 / (2 gigahertz)

= 1 / (2 x 10^9 hertz)

= 0.5 ns

Pipeline Execution Time-

Since there are no stalls in the pipeline, so ideally one instruction is executed per clock cycle. So,

Pipeline execution time

= 1 clock cycle

= 0.5 ns

Speed Up-

Speed up

= Non-pipeline execution time / Pipeline execution time

= 1.6 ns / 0.5 ns

= 3.2

Thus, Option (A) is correct.

Problem-04:

The stage delays in a 4 stage pipeline are 800, 500, 400 and 300 picoseconds. The first stage is replaced with a functionally equivalent design involving two stages with respective delays 600 and 350 picoseconds.

The throughput increase of the pipeline is _____%.

SOL:

Execution Time in 4 Stage Pipeline-

Cycle time

= Maximum delay due to any stage + Delay due to its register

= Max { 800, 500, 400, 300 } + 0

= 800 picoseconds

Thus, Execution time in 4 stage pipeline = 1 clock cycle = 800 picoseconds.

Throughput in 4 Stage Pipeline-

Throughput

= Number of instructions executed per unit time

= 1 instruction / 800 picoseconds

Execution Time in 2 Stage Pipeline-

Cycle time

= Maximum delay due to any stage + Delay due to its register

= Max { 600, 350 } + 0

= 600 picoseconds

Thus, Execution time in 2 stage pipeline = 1 clock cycle = 600 picoseconds.

Throughput in 2 Stage Pipeline-

Throughput

= Number of instructions executed per unit time

= 1 instruction / 600 picoseconds

Throughput Increase-

Throughput increase

= { (Final throughput – Initial throughput) / Initial throughput } x 100

= { (1 / 600 – 1 / 800) / (1 / 800) } x 100

= { (800 / 600) – 1 } x 100

= (1.33 – 1) x 100

= 0.3333 x 100

= 33.33 %