FORMULATION OF LPP PROBLEMS

SIKSHAPATH
WE ARE LIMITLESS

LPP Model Formulation

- Objective function

  a linear relationship reflecting the objective of an operation

  most frequent objective is to *maximize profit* or to *minimize cost.*

- Decision variables

  an unknown quantity representing a decision that needs to be made. It is the quantity the model needs to determine

- Constraint

  a linear relationship representing a restriction on decision making

Steps in Formulating the LP Problems

1. Define the objective. (min or max)
2. Define the decision variables.
3. Write the mathematical function for the objective.
4. Write the constraints.
5. Constraints can be in $\leq$, $=$, or $\geq$ form.

General Formulation of LPP

Max/min $\quad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

subject to:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \ (\leq, =, \geq)\ b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \ (\leq, =, \geq) \ b_2$$

$$:$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ (\leq, =, \geq) \ b_m$$

$$x_1 \geq 0, x_2 \geq 0, \ldots\ldots x_j \geq 0, \ldots\ldots, x_n \geq 0.$$

x_j = decision variables

b_i = constraint levels

c_j = objective function coefficients

a_{ij} = constraint coefficients

Example 1

Two products: Chairs and Tables

Decision: How many of each to make this month?

Objective: Maximize profit

Data

	Tables (per table)	Chairs (per chair)	Hours Available
Profit Contribution	$7	$5	

carpentry	3 hrs	4 hrs	2400
Painting	2 hrs	1 hr	1000

Other Limitations:

- Make no more than 450 chairs
- Make at least 100 tables

Solution

Decision Variables:

$$\mathrm{T} = \text{Num. of tables to make}$$

$$\mathrm{C} = \text{Num. of chairs to make}$$

Objective Function: Maximize Profit

$$\textit{Maximize}\ \$7\,\mathrm{T}\ + \$5\,\mathrm{C}$$

Constraints

Have 2400 hours of carpentry time available

$$3\,\mathrm{T}\ +\ 4\,\mathrm{C} \leq 2400 \qquad \text{(hours)}$$

Have 1000 hours of painting time available

$$2\,\mathrm{T}\ +\ 1\,\mathrm{C} \leq 1000 \qquad \text{(hours)}$$

More Constraints:

Make no more than 450 chairs

$C \leq 450$

Make at least 100 tables

$T \geq 100$

Non negativity:

Cannot make a negative number of chairs or tables

$T \geq 0$

$C \geq 0$

Model

$Max\ 7T + 5C$

Subject to the constraints:

$3T + 4C \leq 2400$

$2T + 1C \leq 1000$

$C \leq 450$

$T \geq 100$

$T, C \geq 0$

Example 2.

Cycle Trends is introducing two new lightweight bicycle frames, the Deluxe and the Professional, to be made from aluminum and steel alloys. The anticipated unit profits are $10 for the Deluxe and $15 for the Professional.

The number of pounds of each alloy needed per frame is summarized on the next slide. A supplier delivers 100 pounds of the aluminum alloy and 80 pounds of the steel alloy weekly. How many Deluxe and Professional frames should Cycle Trends produce each week?

Pounds of each alloy needed per frame

	Aluminum Alloy	Steel Alloy
Deluxe	2	3
Professional	4	2

SOLUTION:

Define the objective

Maximize total weekly profit

Define the decision variables

x_1 = number of Deluxe frames produced weekly

x_2 = number of Professional frames produced weekly

Max $Z = 10x_1 + 15x_2$

Subject To

$2x_1 + 4x_2 \leq 100$

$3x_1 + 2x_2 \leq 80$

$x_1, x_2 \geq 0$

Example 3.

A calculator company produces a handheld calculator and a scientific calculator. Long-term projections indicate an expected demand of at least 150 scientific and 100 handheld calculators each day. Because of limitations on production capacity, no more than 250 scientific and 200 handheld calculators can be made daily.

To satisfy a shipping contract, a minimum of 250 calculators must be shipped each day. If each scientific calculator sold, results in a 20 rupees loss, but each handheld calculator produces a 50 rupees profit; then how many of each type should be manufactured daily to maximize the net profit?

Solution:

Step 1: The decision variables: Since the question has asked for an optimum number of calculators, that's what our decision variables in this problem would be. Let,

$$x = \textit{number of scientific calculators produced}$$
$$y = \textit{number of handheld calculators produced}$$

Step 2: The constraints: Since the company can't produce a negative number of calculators in a day, a natural constraint would be:

$$x \geq 0$$
$$y \geq 0$$

However, a lower bound for the company to sell calculators is already supplied in the problem. We can note it down as:

$$x \geq 150$$
$$y \geq 100$$

We have also been given an upper bound for these variables, owing to the limitations on production by the company. We can write as follows:

$$x \leq 250$$
$$y \leq 200$$

Besides, we also have a joint constraint on the values of *'x'* and *'y'* due to the minimum order on a shipping consignment; given as:

$$x + y \geq 250$$

Step 3: Objective Function: Clearly, here we need to optimize the Net Profit function. The Net Profit Function is given as:

$$P = -20x + 50y$$

Step 4: Solving the problem: the system here –

$$\text{Maximization of } P = -20x + 50y\text{, subject to:}$$
$$150 \leq x \leq 250$$
$$100 \leq y \leq 200$$
$$x + y \geq 250$$