If a coin is tossed 4 times, then the possible outcomes are:

HHHH, HHHT, HHTT, HTTT, THHH, ...

For the longest string of heads, X can take the values 0, 1, 2, 3 and 4.

(As when a coin is tossed 4 times, we can get minimum 0 and maximum 4 strings.)

Now,

$$P(X = 0) = P(0 \text{ head}) = \frac{1}{16}$$
$$P(X = 1) = P(1 \text{ head}) = \frac{4}{16}$$
$$P(X = 2) = P(2 \text{ heads}) = \frac{6}{16}$$
$$P(X = 3) = P(3 \text{ heads}) = \frac{4}{16}$$
$$P(X = 4) = P(4 \text{ heads}) = \frac{1}{16}$$

Thus, the probability distribution of X is given by

X	P(X)
0	$\dfrac{1}{16}$
1	$\dfrac{4}{16}$
2	$\dfrac{6}{16}$
3	$\dfrac{4}{16}$
4	$\dfrac{1}{16}$

Computation of mean and variance

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\dfrac{1}{16}$	0	0
1	$\dfrac{4}{16}$	$\dfrac{4}{16}$	$\dfrac{4}{16}$
2	$\dfrac{6}{16}$	$\dfrac{12}{16}$	$\dfrac{24}{16}$
3	$\dfrac{4}{16}$	$\dfrac{12}{16}$	$\dfrac{36}{16}$
4	$\dfrac{1}{16}$	$\dfrac{4}{16}$	1
		$\sum p_i x_i = 2$	$\sum p_i x_i^2 = 5$

$$\text{Mean} = \sum p_i x_i = 2$$
$$\text{Variance} = \sum p_i x_i 2 - (\text{Mean})^2$$
$$= 5 - 4$$
$$= 1$$