(i) Since $\int_0^1 f(x)\,dx = \int_0^1 6x(1-x)\,dx$ ...(i)

$= 6\left|\frac{x^2}{2} - \frac{x^3}{3}\right|_0^1$

$= 1$, $f(x)$ is a p.d.f

(ii) $F(x) = \begin{cases} 0, & \text{if } x<0 \\ \int_0^x 6t(1-t)\,dt = (3x^2-2x^3), & 0<x\le 1 \\ 1, & \text{if } x>1 \end{cases}$

(iii) $P\left(X \le \frac{1}{2} \mid \frac{1}{3} \le X \le \frac{2}{3}\right)$

$= \frac{P\left(\frac{1}{3} \le X \le \frac{1}{2}\right)}{P\left(\frac{1}{3} \le X \le \frac{2}{3}\right)}$

$= \frac{\int_{1/3}^{1/2} 6x(1-x)\,dx}{\int_{1/3}^{2/3} 6x(1-x)\,dx}$