Fit the curve of the form $y = ab^x$ for

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	126.4

Solution:

The curve to be fitted is $y = ab^x$

taking logarithm on both sides, we get

$\log_{10}(y) = \log_{10}(a) + x\log_{10}(b)$

$Y = A + Bx$ where $Y = \log_{10}(y)$, $A = \log_{10}(a)$, $B = \log_{10}(b)$

which linear in Y,x

So the corresponding normal equations are

$\sum Y = nA + B\sum x$

$\sum xY = A\sum x + B\sum x^2$

The values are calculated using the following table

x	y	$Y = \log_{10}(y)$	x^2	$x \cdot Y$
2	8.3	0.9191	4	1.8382
3	15.4	1.1875	9	3.5626
4	33.1	1.5198	16	6.0793
5	65.2	1.8142	25	9.0712
6	126.4	2.1017	36	12.6105
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$\sum x = 20$	$\sum y = 248.4$	$\sum Y = 7.5424$	$\sum x^2 = 90$	$\sum x \cdot Y = 33.1618$

Substituting these values in the normal equations

$5A + 20B = 7.5424$

$20A + 90B = 33.1618$

Solving these two equations using Elimination method,

$5a + 20b = 7.5424$

and $20a + 90b = 33.1618$

$\therefore 20a + 90b = 33.16$

$5a + 20b = 7.5424 \to (1)$

$20a + 90b = 33.1618 \to (2)$

equation$(1) \times 4 \Rightarrow 20a + 80b = 30.1696$

equation$(2) \times 1 \Rightarrow 20a + 90b = 33.1618$

Substracting $\Rightarrow -10b = -2.9922$

$\Rightarrow 10b = 2.9922$

$\Rightarrow b = \frac{2.9922}{10}$

$\Rightarrow b = 0.29922$

Putting $b = 0.29922$ in equation (1), we have

$5a + 20(0.29922) = 7.5424$

$\Rightarrow 5a = 7.5424 - 5.9844$

$\Rightarrow 5a = 1.558$

$\Rightarrow a = \frac{1.558}{5}$

$\Rightarrow a = 0.3116$

$\therefore a = 0.3116$ and $b = 0.29922$

we obtain $A = 0.3116, B = 0.2992$

$\therefore a = anti\log_{10}(A) = anti\log_{10}(0.3116) = 2.0493$

and $b = anti\log_{10}(B) = anti\log_{10}(0.2992) = 1.9917$

Now substituting this values in the equation is $y = ab^x$, we get

$y = 2.0493 \cdot (1.9917)^x$